Mathematics, Computing and Technology
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Practice quizzes for Units 8 to 10 (Set 1)

Note that the questions in this document are designed for on screen usage via the course website. Some of the instructions within questions may not quite suit this printed version, e.g. when the question refers to 'box below' this should be interpreted as 'box in margin'.

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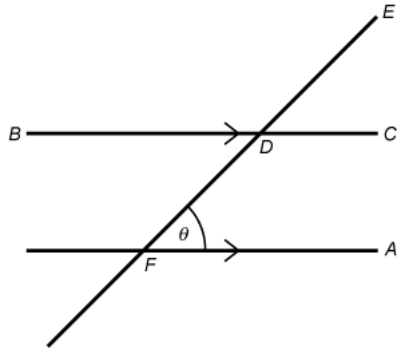
Unit 8 practice quiz questions

Question 1

Select the option that gives an angle equal to θ other than the marked angle in the following diagram.



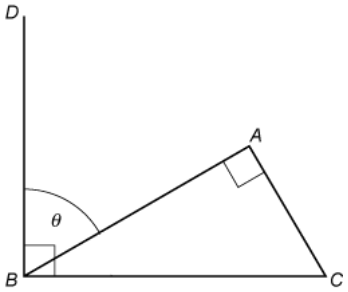
Options

A $\angle EDB$ **B** $\angle AFD$ **C** $\angle CDF$ **D** $\angle AFE$ **E** $\angle EDC$

Answer:

Question 2

Select the option that gives an angle equal to θ other than the marked angle in the following diagram.



Options

A $\angle BAC$ **B** $\angle ABC$ **C** $\angle ACB$ **D** $\angle DBC$ **E** $\angle DBA$

Answer:

Question 3

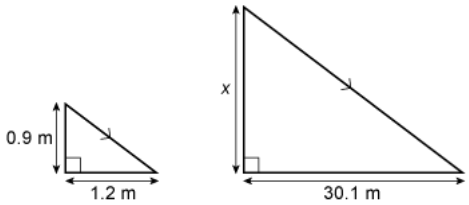
How many lines of symmetry go through the vertices of a regular pentagon (i.e. a regular 5-sided polygon)?

Enter the number in the box below.

Answer:

Question 4

In order to measure the height of a building, a stick was placed perpendicularly in the ground next to the building so that the length of the exposed part of the stick was 0.9 m. The lengths of the shadows of the stick and the building are 1.2 m and 30.1 m respectively. Assume that the rays of light from the sun are parallel and that the building is perpendicular to the ground. This is shown in the diagram below.

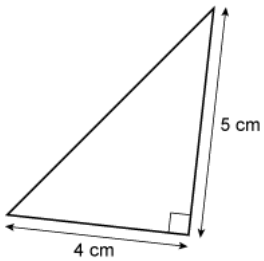


Calculate the height of the building and enter your answer to two decimal places in the box below (do not include units).

Answer:

Question 5

Calculate the length of the third side of the triangle in the following figure.



Enter the length in centimetres in the box below, giving your answer to two significant figures.

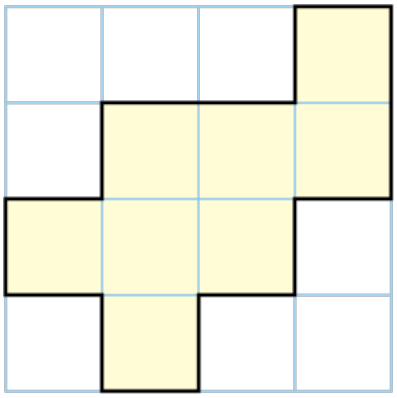
(Your answer should be a number, without units.)

Answer:

Question 6

The wiggliness of a shape with perimeter p and area A is defined by the formula p^2/A.

Calculate the wiggliness of the following shape.



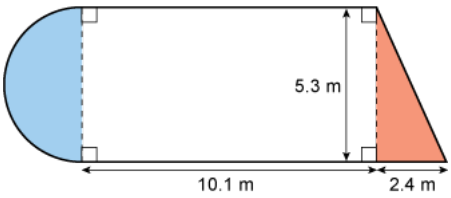
Give your answer in the box below, rounded to two decimal places if necessary.

(Your answer should be a number.)

Answer:

Question 7

The following diagram shows a lawn and you are asked to calculate the length of edging strip to go around it. Any curved sections are semicircles.



Enter your answer for the perimeter in metres (as a number, without units) to two decimal places in the box below.

Answer:

Question 8

Calculate the volume of a cuboid with sides of length 1.2 cm, 2.2 cm, 4.3 cm respectively.

Enter the volume in cubic centimetres in the box below (as a number, without units) to one decimal place.

Answer:

Question 9

Calculate the surface area of a cone of radius 2.1 m and sloping surface of length 3.3 m.

Enter the surface area in square metres in the box below (as a number, without units) to one decimal place.

Answer:

Unit 9 practice quiz questions

Question 1

What is the 26th term of the arithmetic sequence 5, 9, 13, ... ?

(Your answer should be a number.)

Answer:

Question 2

How many terms are there in the arithmetic sequence 2, 6, 10, ..., 22 ?

(Your answer should be a number.)

Answer:

Question 3

What is the sum of the first 27 terms of arithmetic sequence 4, 8, 12, ... ?

(Your answer should be a number.)

Answer:

Question 4

What is the sum of arithmetic sequence 3, 7, 11, ..., 67 ?

(Your answer should be a number.)

Answer:

Question 5

Which of the following expressions is the correct expansion of $(x + 3)(2x + 2)$?

Options

A $2x^2 + 2x + 6$ **B** $2x^2 + 8x - 6$ **C** $2x^2 + 6x + 6$ **D** $2x^2 - 8x + 6$

E $2x^2 + 8x + 6$ **F** $7x + 2$ **G** $2x^2 + 6$

Answer:

Question 6

Expand the brackets in the expression $(3y + 4)^2$, then choose the correct option from the drop-down list for each of the statements below:

The coefficient of y^2 is .....

The coefficient of y is .....

The constant term is .....

Options

A -16 **B** 96 **C** -9 **D** 16 **E** 12 **F** -96

G 0 **H** -12 **I** 24 **J** 9 **K** -24

Question 7

A quiz contestant is asked to work out $34^2 - 32^2$, without using a calculator.

Use a difference of two squares to find two numbers whose product gives the required answer.

What is the **larger** of these two numbers?

Answer:

Question 8

Which **two** of the following equations are quadratic, or can be rearranged into a quadratic equation?

Options

A $2p^2 + 3p = 0$ **B** $2t + 3 = -t + 2$ **C** $\dfrac{1}{2s+3} = \dfrac{2s}{s+3}$

D $\dfrac{t^2}{2t+3} = -\dfrac{1}{t}$ **E** $2y + 3 = 0$

Answer:

Question 9

Factorise $x^2 - 7x + 10$, then choose which **two** of the following are factors of the expression.

Options

A $(x - 10)$ **B** $(x - 3)$ **C** $(x + 1)$ **D** $(x - 2)$

E $(x + 3)$ **F** $(x + 5)$ **G** $(x + 7)$ **H** $(x - 7)$

I $(x + 10)$ **J** $(x + 2)$ **K** $(x - 1)$ **L** $(x - 5)$

Answer:

Question 10

Factorise $3p^2 - 20p + 12$, then choose which **two** of the following are factors of the expression.

Options

A $(3p + 1)$ **B** $(3p + 2)$ **C** $(3p + 6)$ **D** $(p - 6)$

E $(3p - 6)$ **F** $(p - 2)$ **G** $(p + 12)$ **H** $(p + 2)$

I $(3p - 2)$ **J** $(p + 6)$

Answer:

Question 11

Solve $3u^2 - 26u + 16 = 0$, then choose the **two** correct solutions from the list below.

Options

A $2/3$ **B** $-8/3$ **C** $16/3$ **D** $8/3$ **E** $-2/3$ **F** -2

G 2 **H** 6 **I** 8 **J** 10 **K** -8 **L** $-1/3$

Answer:

Question 12

Write the following expression as a single algebraic fraction and choose the correct simplification from the list below.

$$\frac{2}{3x} - \frac{1}{x+1}$$

Options

Answer:

A $\frac{2}{3x}$ **B** $\frac{1}{4x+1}$ **C** $\frac{-3x+2}{x+1}$ **D** $\frac{-x+1}{3x}$

E $\frac{-x+2}{3x(x+1)}$ **F** $\frac{1}{2x-1}$ **G** $\frac{1}{3(x+1)x}$

Question 13

Select the option that is the equation

$$A = \frac{B+1}{C} + 1,$$

rearranged to make B the subject of the equation.

Options

Answer:

A $B = AC - C - 1$ **B** $B = -AC - C - 1$ **C** $B = -AC + C + 1$

D $B = -AC + C - 1$ **E** $B = AC - 1$

Unit 10 practice quiz questions

Question 1

Select the three statements from the list below which are true for the quadratic equation $y = -x^2 + x + 6$.

Options

Answer:

A The graph is a parabola that has no x-intercepts.

B The graph is a parabola that has two x-intercepts.

C The graph is a parabola with a minimum point.

D The graph is a parabola that has one x-intercept.

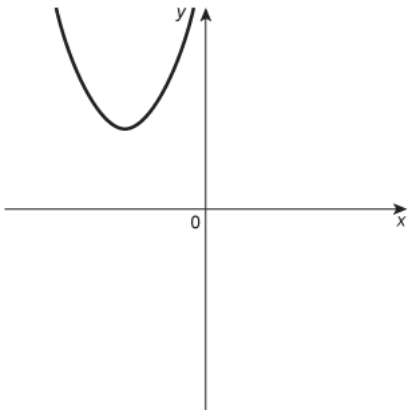
E The point $(0, 6)$ lies on the graph.

F The graph is a parabola with a maximum point.

G The point $(6, 0)$ lies on the graph.

Question 2

Select the quadratic equation from the following list that corresponds to the parabola in the following figure.



Options

Answer:

A $y = x^2 + 4x + 6$

B $y = -x^2 - 4x - 2$

C $y = x^2 - 4x + 6$

D $y = -x^2 - 4x - 6$

E $y = x^2 + 2x - 2$

Question 3

Calculate the x-coordinate of the vertex of the parabola given by

$y = 3x^2 - 66x + 20$.

Enter your answer as a number in the box below.

Answer:

Question 4

Drag the numbers below into the spaces to complete the following sentence.

In ascending numerical order, the solutions of the quadratic equation

$$-4x^2 + 91x + 23 = 0,$$

are $x=$ ☐ and $x=$ ☐.

Options for all boxes

A -0.25 **B** 23 **C** -23 **D** 0.3 **E** -92 **F** 1

Answer: ☐☐

Question 5

Select the option from the list below that gives the solutions to the equation

$x^2 + 2x = 2,$

correct to one decimal place.

Options

A $x = 1.4$ and $x = -0.4$ **B** $x = 0.7$ and $x = -2.7$

C $x = -2.0$ and $x = 0.0$ **D** $x = -1.4$ and $x = 0.4$

E $x = -0.7$ and $x = 2.7$ **F** $x = 2.0$ and $x = 0.0$

Answer: ☐

Question 6

The following equation has one solution that is less than zero.

$x^2 + 2x - 2 = 0$

What is this negative solution?

Enter your answer as a decimal rounded to two decimal places.

Answer: ☐

Question 7

Select the option from the list below that gives the exact solutions to the equation

$x^2 + \frac{2}{3}x - 2 = 0.$

Options

A $x = \frac{1}{3} - \frac{\sqrt{19}}{3}$ and $x = \frac{1}{3} + \frac{\sqrt{19}}{3}$

B $x = -\frac{1}{3} - \frac{\sqrt{17}}{3}$ and $x = -\frac{1}{3} + \frac{\sqrt{17}}{3}$

C $x = -1 - \sqrt{19}$ and $x = -1 + \sqrt{19}$

D $x = 1 - \sqrt{19}$ and $x = 1 + \sqrt{19}$

E $x = -\frac{1}{3} - \frac{\sqrt{19}}{3}$ and $x = -\frac{1}{3} + \frac{\sqrt{19}}{3}$

F $x = \frac{1}{3} - \frac{\sqrt{17}}{3}$ and $x = \frac{1}{3} + \frac{\sqrt{17}}{3}$

Answer: ☐

Question 8

Drag the options below into the spaces to complete the following which puts the quadratic expression $x^2 + 18x + 5$ in completed square form.

$x^2 + 18x + 5 = (x + \square)^2 \square \square$

Options for box 1

A 9 **B** 5 **C** 319 **D** 76 **E** 81 **F** 18

Answer: ☐

Options for box 2

A − **B** +

Answer: ☐

Options for box 3

A 319 **B** 5 **C** 9 **D** 81 **E** 76 **F** 18

Answer: ☐

Question 9

A simple model of the profitability of a company is that the relationship between the profit p when the company manufactures q items is given by

$p = -3q^2 + 68q + 20.$

How many items should the company manufacture to maximise profits?

Enter your answer, as an integer, in the box below.

Answer: ☐

Question 10

The area A of a shape is related to the length x of one side by the quadratic equation:

$A = -3x^2 + 68x + 20.$

What is the maximum area that could be obtained by varying x?

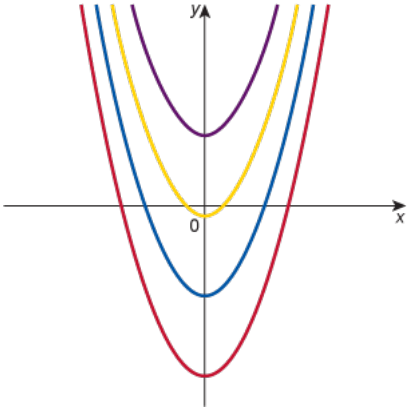
Enter your answer in the box below, without units and rounded to two significant figures.

Answer: ☐

Question 11

The quadratic equation representing a parabola can be written as $y = ax^2 + bx + c$.

Select the statement from the list below that correctly describes which of the coefficients of this equation are changing for the family of parabolas shown below.



Options

Answer:

A a and c are varying whilst b is constant.

B c is changing whilst a and b are constant.

C b and c are varying whilst a is constant.

D b is changing whilst a and c are constant.

E a is changing whilst b and c are constant.

Unit 8 practice quiz solutions

Solution 1

Correct option : E

The given angle $\angle DFA$ and $\angle EDC$ form a pair of corresponding angles, since FA and BC are parallel lines. So the angle equal to θ is $\angle EDC$.

See Unit 8, Subsection 1.2.

Solution 2

Correct option : C

The given angle $\angle DBA$ and $\angle ABC$ add up to $90°$ since $\angle DBC$ is a right angle. So $\angle ABC$ is equal to $90° - \theta$.

In addition $\angle ABC$ and $\angle ACB$ add up to $90°$ since triangle ABC is a right-angled triangle. So $\angle ACB$ is equal to $90° - (90° - \theta) = \theta$.

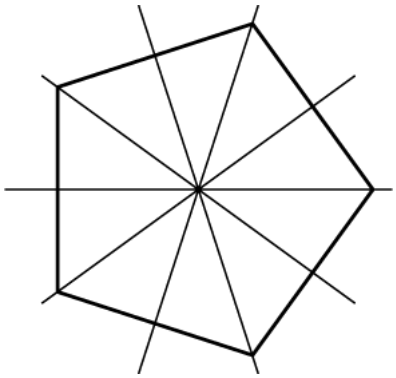
So the angle equal to θ is $\angle ACB$.

See Unit 8, Subsection 2.1.

Solution 3

Correct answer : 5

The lines of symmetry going through the vertices of a regular pentagon are shown below.



Counting the lines in the figure gives 5 lines of symmetry.

See Unit 8, Subsection 2.3.

Solution 4

Correct answer : 22.58

Look at the diagram given in the question. From the right angles and the parallel lines we can deduce that corresponding angles are equal, so the two triangles are similar (this is the AAA case).

Let x be the height of the building.

From the diagram we can also see that the sides representing the lengths of the shadows of the building and the stick correspond and also that the height of the building and the stick correspond. Taking the ratio of corresponding sides gives:

$$\frac{30.1}{1.2} = \frac{x}{0.9}$$

Making x the subject of the equation gives

$$x = \frac{0.9 \times 30.1}{1.2}$$

This evaluates to 22.575, so the answer as 22.58 m to two decimal places.

See Unit 8, Subsection 3.2.

Solution 5

Correct answer : $\boxed{6.4}$

The two given sides are adjacent to the right-angle, so these correspond to sides a and b in the statement of Pythagoras' Theorem. The side to calculate is the hypotenuse of the triangle, which is side c in the statement of Pythagoras' Theorem. Substituting $a = 5$, $b = 4$ into Pythagoras' Theorem gives

$$5^2 + 4^2 = c^2$$

Performing the arithmetic gives:

$$41 = c^2$$

Taking square roots of both sides gives

$$\pm 6.4031 = c$$

Obviously the positive square root is appropriate for the length c, so the answer is that the third side of the triangle is 6.4 cm to two significant figures.

See Unit 8, Subsection 3.3.

Solution 6

Correct answer : $\boxed{32}$

Count the sides of the shape to find the perimeter, which gives a perimeter of 16 units.

Count squares to find the area. In this case the area is 8 square units. The wiggliness of the shape is defined to be the perimeter squared divided by the area, so in this case:

$$\text{wiggliness} = \frac{\text{perimeter}^2}{\text{area}} = \frac{16^2}{8} = 32.$$

So the wiggliness is 32.

See Unit 8, Subsection 4.2.

Solution 7

Correct answer : $\boxed{36.74}$

The question amounts to calculating the perimeter of the lawn. The way to proceed is to calculate the perimeter by adding up the lengths of individual pieces. For the horizontal and vertical sections the lengths are given.

For the semicircular portions the perimeter is obtained by taking half of the circumference of a circle, i.e. $\frac{1}{2} \times 2\pi r = \pi r$. So in this case the perimeter is $\pi \times (5.3/2) = 8.3252$.

For the sloping portions the length is obtained by using Pythagoras' Theorem, i.e. $\sqrt{2.4^2 + 5.3^2} = 5.8181$.

Starting from the bottom left, add together the length to get the perimeter, P, as follows.

$$P = 10.1 + 2.4 + 5.8181 + 10.1 + 8.3252 = 36.7433$$

So the length of edging strip needed is 36.74 m to two decimal places.

See Unit 8, Subsection 4.1 and 4.3.

Solution 8

Correct answer : $\boxed{11.4}$

The formula for the volume of a cuboid with sides w, h and d is whd.

Substituting $w = 1.2$, $h = 2.2$ and $d = 4.3$ into this formula gives:

$$\text{Volume} = 1.2 \times 2.2 \times 4.3$$

Performing the arithmetic gives:

$$\text{Volume} = 11.352$$

So the volume of the cuboid is $11.4\,\text{cm}^3$ to one decimal place.

See Unit 8, Subsection 5.2.

Solution 9

Correct answer : $\boxed{35.6}$

The formula for the surface area of a cone with radius r and sloping surface of length l is $\pi r^2 + \pi rl$.

Substituting $r = 2.1$ and $l = 3.3$ into this formula gives:

$$\text{surface area} = \pi \times 2.1^2 + \pi \times 2.1 \times 3.3$$

Performing the arithmetic gives:

$$\text{surface area} = 35.6257$$

So the surface area of the cone with radius r and sloping surface of length l is $35.6\,\text{m}^2$ to one decimal place.

See Unit 8, Subsection 5.2.

Unit 9 practice quiz solutions

Solution 1

Correct answer : $\boxed{105}$

The nth term of an arithmetic sequence is given by

nth term $= a + (n-1)d$

where a is the first term and d is the difference between successive terms.

In this case, d can be found from the difference of the first two terms: $9 - 5 = 4$.

The 26th term is thus

$$\begin{aligned} 5 + (26-1) \times 4 &= 5 + 25 \times 4 \\ &= 5 + 100 \\ &= 105. \end{aligned}$$

See Unit 9, Subsection 1.1.

Solution 2

Correct answer : $\boxed{6}$

As shown in Unit 9, the number of terms in an arithmetic sequence is given by

$$\frac{L-a}{d} + 1,$$

where L is the last term, a the first term and d the difference between successive terms.

In this case the first term, a, is 2.

The last term, L, is 22.

The difference between terms, d, is $6 - 2 = 4$.

So the number of terms in the sequence is given by,

$$\frac{22-2}{4} + 1 = 6.$$

See Unit 9, Subsection 1.1.

Solution 3

Correct answer : $\boxed{1512}$

The sum, S, of the arithmetic sequence can be calculated using

$$S = \frac{1}{2}n(2a + (n-1)d),$$

where a is the first term and d the difference between successive terms and n the number of terms.

In this case $n = 27$, $a = 4$ and d can be found using $d = 8 - 4 = 4$.

So,

$$\begin{aligned} S &= \frac{1}{2} \times 27 \times (2 \times 4 + (27-1) \times 4) \\ &= \frac{1}{2} \times 27 \times (8 + 104) \\ &= \frac{1}{2} \times 27 \times 112 \\ &= 1512. \end{aligned}$$

Hence the sum of the series is 1512.

See Unit 9, Subsection 1.1.

Solution 4

Correct answer : $\boxed{595}$

The sum, S, of the arithmetic sequence is

$S = 3 + 7 + 11 + \ldots + 67.$

This can be calculated from

$$S = \frac{1}{2}n(a + L),$$

where n is the number of terms in the sequence, a is the first term and L the last.

In this case the first term, a, is 3 and the last term, L, is 67.

To find the number of terms in the sequence, n, we can use the formula

$$\frac{L-a}{d} + 1,$$

where d is the difference between successive terms. In this case $d = 7 - 3 = 4$.

So the number of terms is,

$$\frac{67-3}{4} + 1 = 17.$$

Hence the sum of the series is

$$\frac{1}{2} \times 17 \times (3 + 67) = 595.$$

See Unit 9, Subsection 1.1.

Solution 5

Correct option : $\boxed{\text{E}}$

To multiply two brackets, we multiply each term inside the first bracket with each term inside the second bracket, and add the resulting terms.

$$\begin{aligned} (x + 3)(2x + 2) &= 2x^2 + 2x + 6x + 6 \\ &= 2x^2 + 8x + 6 \end{aligned}$$

See Unit 9, Subsection 2.1.

Solution 6

Matching pairs:

The coefficient of y^2 is	9
The coefficient of y is	24
The constant term is	16

$$\begin{aligned}(3y + 4)^2 &= (3y + 4)(3y + 4)\\ &= 9y^2 + 24y + 16\end{aligned}$$

So,

- the coefficient of y^2 is 9,
- the coefficient of y is 24,
- the constant term is 16.

See Unit 9, Subsection 2.2.

Solution 7

Correct answer : $\boxed{66}$

Using a difference of two squares, $34^2 - 32^2$ can be written as $(34 - 32)(34 + 32)$.

This simplifies to 2×66, so the larger number is 66.

See Unit 9, Subsection 2.3.

Solution 8

Correct option : $\boxed{\text{A}}$

Correct option : $\boxed{\text{C}}$

The following two equations are quadratic:

- $2\,p^2 + 3\,p = 0$, since the highest power of p is 2
- $\frac{1}{2s+3} = \frac{2s}{s+3}$, since multiplying through by $(2\,s + 3)(s + 3)$ and cancelling gives

  $s + 3 = 2\,s(2\,s + 3)$

  and expanding the brackets gives, $s + 3 = 4\,s^2 + 6\,s$ which shows the highest power of s is 2.

The remaining equations are not quadratic, because

- $2\,y + 3 = 0$: the highest power of y is 1.
- $2\,t + 3 = -t + 2$: the highest power of t is 1.
- $\frac{t^2}{2t+3} = -\frac{1}{t}$: this can be re-arranged to give, $-t^3 = 2\,t + 3$ and the highest power of t is 3.

See Unit 9, Subsections 3.1 and 3.2.

Solution 9

Correct option : $\boxed{\text{D}}$

Correct option : $\boxed{\text{L}}$

We need to find a pair of numbers whose product is 10 and sum is -7.

The factor pairs of 10 are 1 and 10; -1 and -10; 2 and 5; -2 and -5 .

The only one of these pairs whose sum is -7 is -2 and -5.

So, $x^2 - 7x + 10 = (x - 2)(x - 5)$.

The required factors are thus $(x - 2)$ and $(x - 5)$.

See Unit 9, Subsection 3.4.

Solution 10

Correct option : D

Correct option : I

The terms in p in the factor brackets must be $3p$ and p, so the factorisation must be of the form $(3p \ldots)(p \ldots)$.

The factor pairs of 12 are 1 and 12; -1 and -12; 2 and 6; -2 and -6; 3 and 4; -3 and -4.

By considering all different possibilities in turn, the factorisation is

$3p^2 - 20p + 12 = (3p - 2)(p - 6)$.

The required factors are thus $(3p - 2)$ and $(p - 6)$.

See Unit 9, Subsection 3.6.

Solution 11

Correct option : A

Correct option : I

First factorise the expression.

The terms in u in the brackets must be $3u$ and u, so the factorisation must be of the form $(3u \ldots)(u \ldots)$.

The factor pairs of 16 are 1 and 16; -1 and -16; 2 and 8; -2 and -8; 4 and 4; -4 and -4.

By considering all different possibilities in turn, the required factorisation is

$3u^2 - 26u + 16 = (3u - 2)(u - 8) = 0$.

So either $3u - 2 = 0$ or $u - 8 = 0$.

Hence, $u = 2/3$ or $u = 8$.

See Unit 9, Subsection 3.6.

Solution 12

Correct option : E

The common denominator is $3\,x\,(x+1)$.

So,

$$\begin{aligned}\frac{2}{3x} - \frac{1}{x+1} &= \frac{2(x+1)}{3x(x+1)} - \frac{1(3x)}{3x(x+1)} \\ &= \frac{2x+2}{3x(x+1)} - \frac{3x}{3x(x+1)} \\ &= \frac{-x+2}{3x(x+1)}.\end{aligned}$$

See Unit 9, Subsection 4.2.

Solution 13

Correct option : A

Starting from the equation

$$A = \frac{B+1}{C} + 1.$$

First subtract 1 from both sides to get

$$A - 1 = \frac{B + 1}{C}.$$

Multiplying by C gives

$$C\,(A - 1) = B + 1.$$

Expanding brackets gives

$$AC - C = B + 1.$$

Making B the subject of this equation gives

$$B = AC - C - 1.$$

See Unit 9, Subsection 5.2.

Unit 10 practice quiz solutions

Solution 1

Correct option : B

Correct option : E

Correct option : F

The coefficient of x^2 is negative so the parabola has a maximum point.

The point $(0, 6)$ lies on the graph, i.e. the y-intercept is 6.

The discriminant of the equation is

$$\begin{aligned} b^2 - 4 \times a \times c &= (1)^2 - 4 \times (-1) \times (6) \\ &= 25 \end{aligned}$$

As the discriminant is positive the parabola has two x-intercepts.

See Unit 10, Subsections 2.2, 2.3 and 3.3.

Solution 2

Correct option : A

The parabola shown is u-shaped, so the coefficient of x^2 is positive.

Judging by the portion of the parabola that is visible, we can see that the y-intercept of the given parabola must be positive so the constant term is positive.

The given parabola has no x-intercepts so the discriminant of the quadratic must be negative.

The vertex of the parabola lies to the left of the y-axis, so the x-coordinate of the vertex (given by $-\frac{b}{2a}$) must be negative.

Looking at the given equations the only one that satisfies these criteria is $y = x^2 + 4x + 6$.

See Unit 10, Subsections 2.3 and 3.3.

Solution 3

Correct answer : 11

The x-coordinate of the vertex of the parabola represented by the quadratic equation $y = ax^2 + bx + c$ is $x = -\frac{b}{2a}$.

In this case the quadratic equation $y = 3x^2 - 66x + 20$ has $a = 3$, $b = -66$ and $c = 20$, so the x-coordinate of the vertex is given by

$$-\frac{b}{2a} = -\frac{(-66)}{2 \times (3)} = 11.$$

So the required answer is 11.

See Unit 10, Subsection 2.4.

Solution 4

Correct option for box 1: B

Correct option for box 2: D

The quadratic equation

$-4\,x^2 + 91\,x + 23 = 0,$

has coefficients $a = -4$, $b = 91$ and $c = 23$.

Using the quadratic formula the solutions are:

$$x = \frac{-(91) \pm \sqrt{(91)^2 - 4 \times (-4) \times (23)}}{2 \times (-4)}$$

$$= \frac{-91 \pm \sqrt{8649}}{-8}$$

$$= \frac{-91 \pm 93}{-8}$$

$$= -0.25 \text{ or } 23.$$

So the completed sentence should read:

"In ascending numerical order, the solutions of the quadratic equation

$-4x^2 + 91x + 23 = 0,$

are x=−0.25 and x=23. "

See Unit 10, Subsection 3.2.

Solution 5

Correct option : B

First re-arrange the given equation

$x^2 + 2x = 2,$

into the form $ax^2 + bx + c = 0$, giving

$x^2 + 2x - 2 = 0,$

From this form we see that the coefficients are $a = 1$, $b = 2$ and $c = -2$.

Using the quadratic formula the solutions are:

$$x = \frac{-(2) \pm \sqrt{(2)^2 - 4 \times (1) \times (-2)}}{2 \times (1)}$$

$$= \frac{-2 \pm \sqrt{12}}{2}$$

$$= 0.7 \text{ or } -2.7 \text{ to 1 d.p.}$$

So the solutions are $x = 0.7$ and $x = -2.7$.

See Unit 10, Subsection 3.2.

Solution 6

Correct answer : -2.73

The quadratic

$x^2 + 2\,x - 2 = 0$, has coefficients $a = 1$, $b = 2$ and $c = -2$.

Using the quadratic formula the solutions are:

$$x = \frac{-(2) \pm \sqrt{(2)^2 - 4 \times (1) \times (-2)}}{2 \times (1)}$$

$$= \frac{-2 \pm \sqrt{12}}{2}$$

$$= 0.732050808 \text{ or } -2.732050808.$$

So the negative solution to the quadratic is -2.73 to 2 d.p.

See Unit 10, Subsection 3.2.

Solution 7

Correct option : E

First clear the fractions in the given equation

$$x^2 + \frac{2}{3}x - 2 = 0,$$

by multiplying by 3 to obtain

$$3\,x^2 + 2\,x - 6 = 0,$$

Now use the the quadratic formula with parameters $a = 3$, $b = 2$ and $c = -6$:

$$\begin{aligned} x &= \frac{-(2) \pm \sqrt{(2)^2 - 4 \times (3) \times (-6)}}{2 \times (3)} \\ &= \frac{-2 \pm \sqrt{76}}{6} \\ &= \frac{-2}{6} \pm \frac{2\sqrt{19}}{6} \\ &= -\frac{1}{3} - \frac{\sqrt{19}}{3} \text{ or } -\frac{1}{3} + \frac{\sqrt{19}}{3}. \end{aligned}$$

So the exact solutions to the quadratic equation are $x = -\frac{1}{3} - \frac{\sqrt{19}}{3}$ and $x = -\frac{1}{3} + \frac{\sqrt{19}}{3}$.

See Unit 10, Subsection 3.2.

Solution 8

Correct option for box 1: A

Correct option for box 2: A

Correct option for box 3: E

Complete the square as follows.

First note that the coefficient of x is 18, half of which is 9.

So we can write $x^2 + 18\,x = (x + 9)^2 - (9)^2$

and then,

$$\begin{aligned} x^2 + 18\,x + 5 &= (x + 9)^2 - (9)^2 + 5 \\ &= (x + 9)^2 - 76 \end{aligned}$$

So the completed square form is $(x + 9)^2 - 76$.

See Unit 10, Subsection 4.2.

Solution 9

Correct answer : 11

The maximum value of an n-shaped parabola occurs at the vertex, so we now proceed to locate the vertex.

The x-coordinate of the vertex of the quadratic expression $ax^2 + bx + c$ is $x = -b/(2\,a)$.

In this case the expression $-3q^2 + 68q + 20$ is a quadratic in the variable q and the coefficients are $a = -3$, $b = 68$ and $c = 20$. So the x-coordinate of the vertex is given by

$$-\frac{b}{2 \times a} = -\frac{(68)}{2 \times (-3)} = 11.33333333.$$

The number of items is an integer, so rounding to the nearest integer, 11 items are needed to maximise the profit.

See Unit 10, Subsection 5.2.

Solution 10

Correct answer : $\boxed{410}$

The maximum value of an n-shaped parabola occurs at the vertex, so we now proceed to locate the vertex.

The x-coordinate of the vertex of the quadratic $ax^2 + bx + c$ is $x = -b/(2\,a)$.

In this case the quadratic expression $-3x^2 + 68x + 20$ has coefficients $a = -3$, $b = 68$ and $c = 20$. So the x-coordinate of the vertex is given by

$$-\frac{b}{2 \times a} = -\frac{(68)}{2 \times (-3)} = \frac{34}{3}.$$

The maximum area obtained is the value of the quadratic at this point, which is

$$\begin{aligned} A &= -3 \times \left(\frac{34}{3}\right)^2 + 68 \times \left(\frac{34}{3}\right) + 20 \\ &= -\frac{1156}{3} + \frac{2312}{3} + 20 \\ &= \frac{1216}{3} \end{aligned}$$

Rounding this to two significant figures gives the answer 410.

See Unit 10, Subsection 5.2.

Solution 11

Correct option : $\boxed{\text{B}}$

Changing the coefficient a changes how narrow the parabola is. If b is not equal to zero, that is, if the axis of symmetry does not lie on $x = 0$, changing a also shifts the parabola both horizontally and vertically.

In this case all of the parabolas are the same shape, so a is constant.

Changing the coefficient b moves the parabola both horizontally and vertically. Unless the other coefficients are also changed, the y-intercept remains fixed.

Both a and b affect the position of the vertex, since its x-coordinate is given by $x = -b/(2a)$.

In this case, since all the vertices have an x-coordinate of zero, b must be zero and is thus not changing.

Changing the coefficient c shifts the parabola vertically, thus changing the y-intercept.

In this case the y-intercept is changing, and since a and b are constant, c must be changing.

So the correct statement is:

c is changing whilst a and b are constant.

See Unit 10, Subsection 2.2.